STUDENT'S NAME: ______________________________

TEACHER'S NAME: ______________________________

2022

HURLSTONE AGRICULTURAL HIGH SCHOOL

HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using a black pen
- NESA approved calculators may be used
- A reference sheet is provided in the Section I booklet
- For questions in Section II, use the relevant booklet for writing your solutions.
- This examination paper is not to be removed from the examination centre

Total marks: 70

Section I – 10 marks (pages 2 – 4)

- Attempt Questions 1 – 10. The multiple-choice answer sheet has been provided
- Allow about 15 minutes for this section

Section II – 60 marks (pages 5 – 10)

- Attempt Questions 11 – 14, write your solutions in the spaces provided.
- You have been provided with 4 separate answer booklets, one for each question.
- Extra working pages are available if required.
- Allow about 1 hours and 45 minutes for this section.

SECTION I (10 marks)

Attempt Questions 1 - 10

Allow about 15 minutes on this section.

Use the multiple-choice answer sheet provided for Questions 1 – 10.

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1. Two particles oscillate horizontally. The displacement of the first is given by $x = 3\sin 4t$ and the displacement of the second is given by $x = a\sin nt$. In one oscillation, the second particle covers twice the distance of the first particle, but in half the time.

   What are the values of a and n?

   (A) $a = 1.5, \quad n = 2$

   (B) $a = 1.5, \quad n = 8$

   (C) $a = 6, \quad n = 2$

   (D) $a = 6, \quad n = 8$

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2. Which of the following is the expression for the rate of change of the area of a circle (A) with respect to its radius (r)?

   (A) $\dfrac{dr}{dt} = 2\pi r \times \dfrac{dA}{dt}$

   (B) $\dfrac{dr}{dt} = 2\pi r \times \dfrac{dA}{dr}$

   (C) $\dfrac{dA}{dr} = 2\pi r \times \dfrac{dt}{dr}$

   (D) $\dfrac{dA}{dt} = 2\pi r \times \dfrac{dr}{dt}$

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3. A particle is moving in a straight line such that its displacement (x metres) from the origin after t seconds is given by $x = \sin^2 t$.

   Which of the following best describes the motion of the particle when $t = \dfrac{2\pi}{3}$?

   (A) The particle is moving to the right with increasing speed.

   (B) The particle is moving to the right with decreasing speed.

   (C) The particle is moving to the left with increasing speed.

   (D) The particle is moving to the left with decreasing speed.

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4. For a polynomial $P(x) = x^3 - 3x^2 + 1$ with roots α, β and γ it is known that:

   $$\alpha + \beta + \gamma = 3 \quad \text{and} \quad \alpha^2 + \beta^2 + \gamma^2 = 9$$

   What is the value of $\alpha^3 + \beta^3 + \gamma^3$?

   (A) 3

   (B) 10

   (C) 24

   (D) 27

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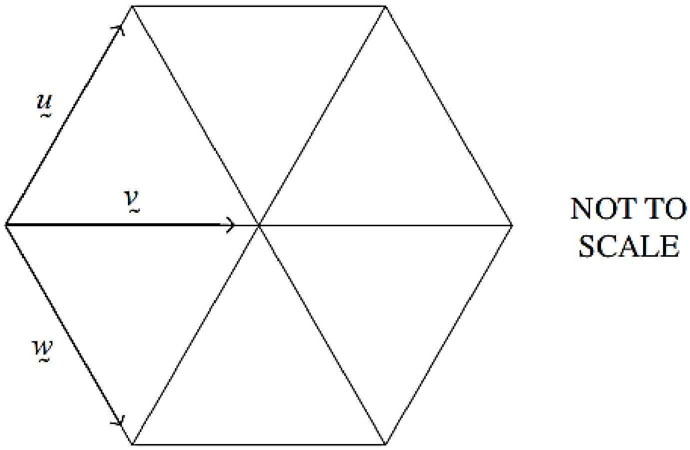
5. Six equilateral triangles form a hexagon with side lengths of 4 cm. The vectors $\underset{\sim}{u}$, $\underset{\sim}{v}$ and $\underset{\sim}{w}$ are shown in the diagram.



Which of the following is the value of $\underset{\sim}{u} \cdot (\underset{\sim}{u} + \underset{\sim}{v} + \underset{\sim}{w})$?

(A) 8 (B) 16

(C) 32 (D) 48

6. Which expression describes the probability of k "3s" being rolled on 20 successive tolls of a fair six-sided die?

(A) $\binom{20}{k}\left(\frac{1}{6}\right)^k\left(\frac{5}{6}\right)^{20-k}$ (B) $\binom{20}{k}\left(\frac{3}{6}\right)^k\left(\frac{3}{6}\right)^{20-k}$

(C) $\binom{20}{k}\left(\frac{5}{6}\right)^k\left(\frac{1}{6}\right)^{20-k}$ (D) $\binom{20}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^{17}$

7. The expression $2\cos x - 3\sin x$ is written in the form $R\cos(x+\theta)$, where $R > 0$ and $0 \le \theta \le \frac{\pi}{2}$.

Which of the following is the value of $\tan\theta$?

(A) $-\frac{3}{2}$ (B) $-\frac{2}{3}$

(C) $\frac{3}{2}$ (D) $\frac{2}{3}$

8. Which of the following is the derivative of $y = \tan^{-1}\left[2f(x)\right]$?

(A) $\dfrac{dy}{dx} = \dfrac{1}{1+\left[f(x)\right]^2}$

(B) $\dfrac{dy}{dx} = \dfrac{2}{1+4\left[f(x)\right]^2}$

(C) $\dfrac{dy}{dx} = \dfrac{f'(x)}{1+4\left[f(x)\right]^2}$

(D) $\dfrac{dy}{dx} = \dfrac{2f'(x)}{1+4\left[f(x)\right]^2}$

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9. The definite integral $\displaystyle\int_e^{e^2} \frac{2}{x(\log_e x)^2}\,dx$, is evaluated using the substitution $u = \log_e x$.

Which of the following gives the correct value of the integral?

(A) $2\left(\dfrac{1}{e} - \dfrac{1}{e^2}\right)$

(B) $2\left(\dfrac{1}{e^2} - \dfrac{1}{e}\right)$

(C) 1

(D) -1

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10. A Mathematics department consists of 12 teachers. Nine of the teachers wear glasses and three of the teachers do not wear glasses.

Five of these teachers go out to dinner together.

In how many ways will there be more teachers who wear glasses than teachers who do not wear glasses in the group who go out for dinner?

(A) 126

(B) 220

(C) 756

(D) 792

~ End of Section 1 ~

SECTION II

60 marks

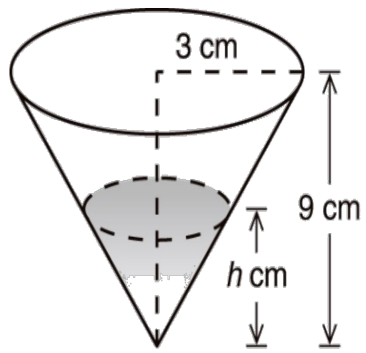
Attempt Questions 11 - 14

Allow about 1 hours and 45 minutes on this section.
For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.
Answer each question in a separate answer booklet.

Question 11 (15 marks) Use the Question 11 BOOKLET **Marks**

(a) The figure shows an inverted right conical vessel. The base radius and the height of the vessel are 3 cm and 9 cm respectively.

Water is poured into the vessel at a constant rate of 2 cm^3/s.



i) Let V cm^3 be the volume of water in the vessel when the depth of water is h cm. Express V in terms of h. **2**

ii) When the depth of water is 6 cm, find the rate of change of the area of the water surface with respect to time. **3**

(b) After a body dies, the percentage P of radioactive carbon-14 in the bones after t years is given according to the function $P = 100e^{-0.00012t}$.

What is the half-life of carbon-14, correct to 2 decimal places? **2**

Question 11 continued on next page ….

Question 11 (15 marks) continued … **Marks**

(c) A man was stabbed to death in a street.
The coroner arrived at the scene at 11p.m.
She took the temperature of the body t hours after death and found it to be $25°C$.
She took the temperature again 1 hour later, $t+1$ hours after death, and found it to be $20°C$.
Suppose the air temperature that night was $16°C$ and the law of cooling of the body is given by $T - T_0 = (37.5 - T_0)e^{-kt}$,
where T_0 is the air temperature and T is the temperature of the body t hours after death.

i) Find the value of the constant k correct to 2 significant figures. **2**

ii) When was the murder committed correct to the nearest hour? **2**

(d) Prove, by mathematical induction, that for all integers $n \geq 1$,

$$1 + 2\times 2^{-1} + 3\times 2^{-2} + 4\times 2^{-3} + \ldots + n\times 2^{-(n-1)} = \frac{2^{n+1} - n - 2}{2^{n-1}}$$ **4**

~ End of Question 11 ~

2022 Mathematics Extension 1 Trial Examination Section II

Question 12 (15 marks) Use the Question 12 BOOKLET **Marks**

(a) Let $P(x) = x^3 + ax^2 - x + 1$ be a polynomial where a is a real number. **2**
When $P(x)$ is divided by $x - 2$ the remainder is 15.
Find the remainder when $P(x)$ is divided by $x + 3$.

(b) Write the expansion of $(2x - y)^5$. **2**

(c) i) Find the general term of $\left(x + \frac{1}{x}\right)^8$. **1**

ii) Find the coefficient of x^2 in $\left(x + \frac{1}{x}\right)^8$. **2**

(d) Resolve the vector $\underset{\sim}{a}$ into component form $\underset{\sim}{a} = x\underset{\sim}{i} + y\underset{\sim}{j}$, given $\underset{\sim}{a}$ has a magnitude of 6 units and has a direction of $50°$ to the positive x-axis.

Give answers correct to two decimal places. **2**

(e) Given $\underset{\sim}{c} = 3\underset{\sim}{i} - 6\underset{\sim}{j}$, find vector $\underset{\sim}{d}$ of magnitude 5 in the direction of $\underset{\sim}{c}$. **2**

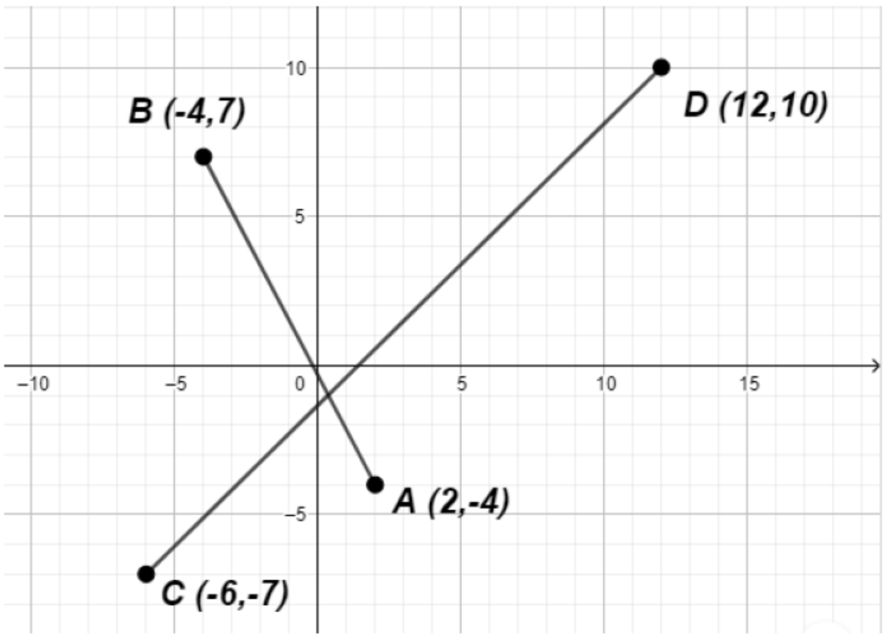
Question 12 continued on page 8 …

Question 12 (15 marks) continued … **Marks**

(f)

i) Explain **in words** only why $\underset{\sim}{i} \cdot \underset{\sim}{j} = 0$? **1**

ii) The diagram below shows the lines AB and CD.
The line AB passes through the points $(2, -4)$ and $(-4, 7)$ and the line CD passes through $(-6, -7)$ and $(12, 10)$.
Use vector methods to find the acute angle between the two lines.
Write your answer to the nearest degree. **3**



~ End of Question 12 ~

Question 13 (15 marks) Use the Question 13 BOOKLET **Marks**

(a) A multiple-choice test contains ten questions. Each question has four choices for the correct answer. Only one of the choices is correct.

i) What is the probability of getting 90% with random guessing? **1**

ii) What is the probability of getting at most 90% with random guessing? **2**

(b) The probability that the 7:30 a.m. train arrives on time is 0.85.
Find an expression for the probability that the train is on time at least five days during one week. **2**

(c) Find the exact value of $\sin\left[2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$. **1**

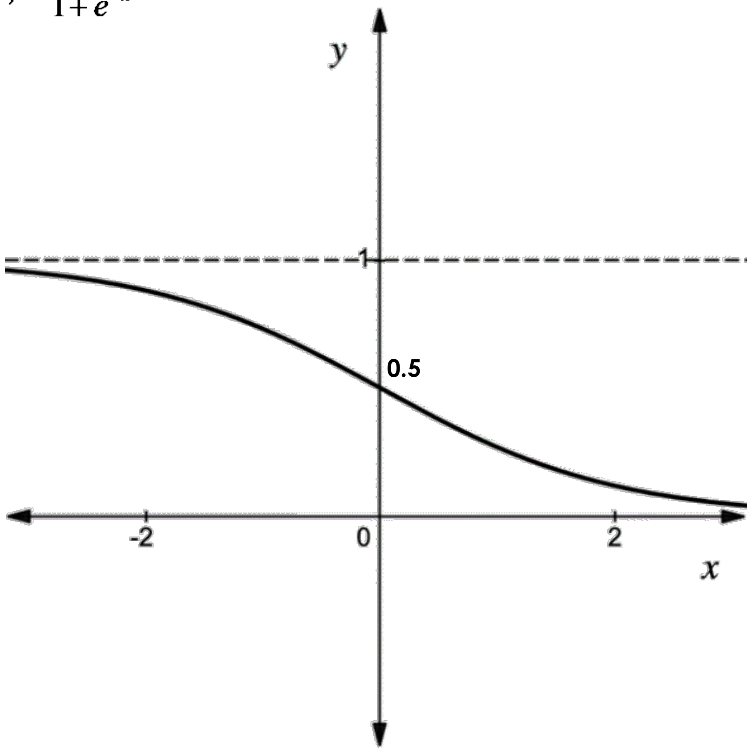
(d) Find the maximum value of $y = \sqrt{3}\cos x + \sin x$. **2**

(e) Use t-formulae to solve the equation $\cos x - \sin x = 1$, where $0 \le x \le 2\pi$. **3**

Question 13 continued on next page ….

Question 13 (15 marks) continued … **Marks**

(f) The graph of $y = f(x) = \dfrac{e^{-x}}{1+e^{-x}}$ is shown.



i) Find the equation of $y = f^{-1}(x)$. **2**

ii) Sketch $y = f^{-1}(x)$, showing any asymptotes and intercepts with the coordinate axes. **2**

~ End of Question 13 ~

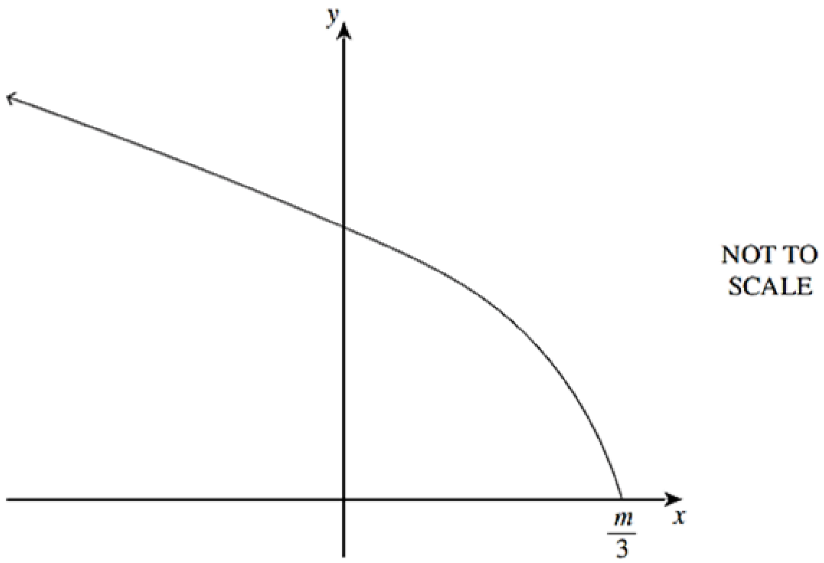
Question 14 (15 marks) Use the Question 14 BOOKLET **Marks**

(a) Use the substitution $u = t + 1$ or otherwise to evaluate $\int_0^1 \frac{t}{\sqrt{t+1}}\,dt$.

Leave your answer in exact form. **3**

(b) Solve the differential equation $\frac{dy}{dx} = \frac{\cos^2 y}{e^x}$ given that $y(0) = \frac{\pi}{4}$.
Express your answer in the form $y = f(x)$. **3**

(c) Let $f(x) = \sqrt{m - 3x}$ for $x \le \frac{m}{3}$. The graph of $f(x)$ is shown.



The area enclosed by the graph , the x-axis and y-axis is rotated about the y-axis.

Find the value of m such that the volume of the solid formed is $\frac{5000\pi}{27}$ units^3. **4**

(d) i) Show that $\frac{d}{dx}\left(-\cot x\right) = \frac{1}{\sin^2 x}$. **2**

ii) Use the substituion $x = 4\sin\theta$ and the result from (i) to

show that $\int_2^{2\sqrt{3}} \frac{1}{x^2\sqrt{16 - x^2}}\,dx = \frac{\sqrt{3}}{24}$. **3**

~ End of Question 14 ~

~ End of Trial Examination ~

2022 Y12 Trial Maths Ext 1		
Question MC	Solutions and Marking Guidelines	
	Outcomes Addressed in this Question	
Applications of Differentiation MA-C3, Graphing Techniques (F2) (C3.1 The first and second derivatives; C3.2 Optimisation and motion		
Question	**Solutions**	**Marking Guidelines**
1	D	
2	D	
3	C	
4	C	
5	$\underset{\sim}{u}+\underset{\sim}{v}+\underset{\sim}{w}=2\underset{\sim}{v}$. $\therefore \underset{\sim}{u}\cdot(\underset{\sim}{u}+\underset{\sim}{v}+\underset{\sim}{w})=\underset{\sim}{u}\cdot 2\underset{\sim}{v}$ $\cos\theta=\dfrac{\underset{\sim}{u}\cdot 2\underset{\sim}{v}}{\lvert\underset{\sim}{u}\rvert\lvert 2\underset{\sim}{v}\rvert}$ $\therefore \underset{\sim}{u}\cdot 2\underset{\sim}{v}=\lvert\underset{\sim}{u}\rvert\lvert 2\underset{\sim}{v}\rvert\cos\theta$ Since all the triangles are equilateral, $\theta=\dfrac{\pi}{3}$, $\lvert\underset{\sim}{u}\rvert=4$ and $\lvert 2\underset{\sim}{v}\rvert=8$. $\therefore \underset{\sim}{u}\cdot 2\underset{\sim}{v}=4\times 8\times\cos\dfrac{\pi}{3}$ $=16$ B	
6	A	
7	$2\cos x-3\sin x=R\cos(x+\theta)$ $=R\cos x\cos\theta-R\sin x\sin\theta$ Equating both sides: $2\cos x=R\cos x\cos\theta$ and $3\sin x=R\sin x\sin\theta$ Therefore: $R\cos\theta=2 \quad (1)$ $R\sin\theta=3 \quad (2)$ $\dfrac{(2)}{(1)}$ $\therefore \tan\theta=\dfrac{3}{2}$ C	
8	If $y=\tan^{-1}\left[2f(x)\right]$ $\dfrac{dy}{dx}=\dfrac{1}{1+\left[2f(x)\right]^2}\times 2f'(x)$ $=\dfrac{2f'(x)}{1+4\left[f(x)\right]^2}$ D	

9	$\int_{e}^{e^2} \frac{2}{x(\log_e x)^2}dx = 2\int_{e}^{e^2} \frac{1}{(\log_e x)^2} \times \frac{1}{x}dx$ $= 2\int_{1}^{2} \frac{1}{u^2}du$ $= 2\int_{1}^{2} u^{-2}du$ $= 2\left[\frac{u^{-1}}{-1}\right]_1^2$ $= -2\left[\frac{1}{u}\right]_1^2$ $= -2\left(\frac{1}{2}-1\right)$ $= 1$ C	
10	5 glasses: $\binom{9}{5}$ 4 glasses, 1 no glasses: $\binom{9}{4} \times \binom{3}{1}$ 3 glasses, 2 no glasses: $\binom{9}{3} \times \binom{3}{2}$ $\text{total} = \binom{9}{5} + \binom{9}{4} \times \binom{3}{1} + \binom{9}{3} \times \binom{3}{2}$ $= 756$ C	

2022 Y12 Trial Maths Ext 1

Question MC Solutions and Marking Guidelines

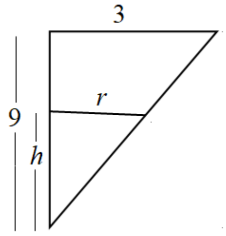
Outcomes Addressed in this Question

ME12-1 applies techniques involving proof or calculus to model and solve problems

Outcome	**Solutions**	**Marking Guidelines**

(a) i) Let r cm be the radius of water in the vessel.

$$\frac{r}{h} = \frac{3}{9}$$
$$r = \frac{1}{3}h$$
$$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h$$
$$= \frac{1}{27}\pi h^3$$



2 marks for correct solution

1 mark for showing the ratio of r:h

(a) ii) Differentiate both sides with respect to time t, we have

$$\frac{dV}{dt} = \frac{1}{27}\pi \cdot 3h^2 \cdot \frac{dh}{dt}$$
$$= \frac{1}{9}\pi h^2 \cdot \frac{dh}{dt}$$

$\because$ Water is poured into the vessel at a constant rate of 2 cm^3/s.

$$\therefore \quad \frac{dV}{dt} = 2$$

When $h = 6$,

$$2 = \frac{1}{9}\pi(6)^2 \cdot \frac{dh}{dt}$$
$$\frac{dh}{dt} = \frac{1}{2\pi}$$

Let S cm^2 be the area of the water surface.

$$S = \pi r^2$$
$$= \pi\left(\frac{1}{3}h\right)^2$$
$$= \frac{1}{9}\pi h^2$$

Or use $\frac{dS}{dt} = \frac{dS}{dh} \times \frac{dh}{dV} \times \frac{dV}{dt}$

$$\frac{dS}{dt} = \frac{1}{9}\pi\left(2h \cdot \frac{dh}{dt}\right)$$
$$= \frac{2}{9}\pi h \cdot \frac{dh}{dt}$$

When $h = 6$,

$$\frac{dS}{dt} = \frac{2}{9}\pi(6)\left(\frac{1}{2\pi}\right)$$
$$= \frac{2}{3}$$

$\therefore$ The area of the water surface is increasing at a rate of $\frac{2}{3}$ cm^2/s.

3 marks for correct solution

1 mark for correct derivative of V' or h'

1 mark for correct S' with sub.

b) The half-life is the time at which P = 50.

$$50 = 100e^{-0.00012t}$$
$$0.5 = e^{-0.00012t}$$
$$\ln 0.5 = -0.00012t$$
$$t = \frac{\ln 0.5}{-0.00012}$$
$$= 5776.23 \text{ years}$$

2 marks for correct solution

1 mark for some progress

c) i) Suppose the man has died for t_o hours at 11 p.m.
Now $T_o = 16$

$$\therefore 25 - 16 = (37.5 - 16)e^{-kt_o}$$
$$9 = 21.5e^{-kt_o} \qquad (1)$$
$$When\ t = t_o + 1 \quad T = 20$$
$$20 - 16 = (37.5 - 16)e^{-k(t_o+1)}$$

When t = t_o, T=25 $\qquad 4 = 21.5e^{-k(t_o+1)} \qquad (2)$

$$\frac{(2)}{(1)} \quad \frac{4}{9} = e^{-kt_o - k + kt_o}$$
$$= e^{-k}$$
$$-k = \ln\frac{4}{9}$$
$$k = 0.81$$

2 marks for correct solution

1 mark for some progress

(c)ii) Sub $e^{-k} = \frac{4}{9}$ in (1)

$$9 = 21.5(\frac{4}{9})^{t_o}$$
$$\frac{9}{21.5} = (\frac{4}{9})^{t_o}$$
$$\ln\frac{9}{21.5} = t_o \ln(\frac{4}{9}) \text{ in (1)}$$
$$t_o = \frac{\ln\frac{9}{21.5}}{\ln(\frac{4}{9})}$$
$$= 1.074$$

The man has died for 1 hour at 11 p.m.
The murder was committed at 10 p.m.

2 marks for correct solution

1 mark for some progress

d)

$$1+2\times 2^{-1}+3\times 2^{-2}+4\times 2^{-3}+\ldots+n\times 2^{-1(n-1)}=\frac{2^{n+1}-n-2}{2^{n-1}}$$

4 marks for correct solution

For $n=1$ $\quad \text{LHS}=1+2\times 2^{-1} = 1$ $\quad \text{RHS}=\dfrac{2^{1+1}-1-2}{2^{1-1}} = 1$

$\therefore$ Statement is true for $n = 1$

1 mark for $n = 1$ and $n = k$ correctly

Assume statement true for $n = k$, that is,

$$1+2\times 2^{-1}+3\times 2^{-2}+4\times 2^{-3}+\ldots+k\times 2^{-1(k-1)}=\frac{2^{k+1}-k-2}{2^{k-1}}$$

Prove true for $n = k+1$

That is, we show that

$$1+2\times 2^{-1}+3\times 2^{-2}+4\times 2^{-3}+\ldots+k\times 2^{-1(k-1)}+(k+1)\times 2^{-1(k+1-1)}=\frac{2^{k+1+1}-(k+1)-2}{2^{k+1-1}}$$

2 marks for proves inductive step by assuming true for $n = k+1$

$$\begin{aligned}
LHS &= \frac{2^{k+1}-k-2}{2^{k-1}}+(k+1)\times 2^{-1(k+1-1)} \\
&= \frac{2^{k+1}-k-2}{2^{k-1}}+(k+1)\times 2^{-1(k+1-1)} \\
&= \frac{2^{k+1}-k-2}{2^{k-1}}+\frac{k+1}{2^{k}} \\
&= \frac{2^{k+2}-2k-4}{2^{k}}+\frac{k+1}{2^{k}} \qquad *(\times\frac{2}{2}) \\
&= \frac{2^{k+2}-k-3}{2^{k}} \\
&= \text{RHS}
\end{aligned}$$

$\therefore$ By the principle of mathematical induction the statement is true for $n\geq 1$

Year 12	Mathematics Extension 1	Assessment Task 4 2022
Question No. 12	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

ME11-2 manipulates algebraic expressions and graphical functions to solve problems.
ME12-2 applies concepts and techniques involving vectors and projectiles to solve problems.
ME12-5 applies appropriate statistical processes to present, analyse and interpret data.

Part	Solutions	Marking Guidelines
(a) **ME11-2**	$P(x) = x^3 + ax - x + 1$ $P(2) = 15$ $15 = (2)^3 + a(2)^2 - (2) + 1$ $= 8 + 4a - 2 + 1$ $4a = 8$ $a = 2$ $\therefore P(x) = x^3 + 2x^2 - x + 1$ $P(-3) = x^3 + 2x^2 - x + 1$ $= (-3)^3 + 2(-3)^2 - (-3) + 1$ $= -27 + 18 + 3 + 1$ $= -5$	**Award 2** marks for the correct solution. **Award 1** mark for finding $a = 2$.
(b) **ME12-5**	$(2x - y)^5 = (2x)^5 - {}^5C_1(2x)^4 y + {}^5C_2(2x)^3 y^2 - {}^5C_3(2x)^2 y^3 + {}^5C_4(2x)y^4 - y^5$ $= 32x^5 - 80x^4 + 80x^3y^2 - 40x^2y^3 + 10xy^4 - y^5$	**Award 2** marks for the correct solution. **Award 1** mark for substantial progress towards the solution.
(c) (i) **ME12-5**	${}^8C_r(x)^{8-r}\left(\frac{1}{x}\right)^r = {}^8C_r(x)^{8-2r}$	**Award 1** mark for the correct solution.
(ii) **ME12-5**	For the term with x^2 $8 - 2r = 2$ $r = 3$ The term is: ${}^8C_3(x)^{8-3}\left(\frac{1}{x}\right)^3 = 56x^2$ The coefficient of x^2 is 56	**Award 2** marks for the correct solution. **Award 1** mark for substantial progress towards the solution.

(d) **ME12-2**	$\lvert \underset{\sim}{a} \rvert = 6$ and $\theta = 50^\circ$ $\underset{\sim}{a} = \lvert \underset{\sim}{a} \rvert \cos\theta\, \underset{\sim}{i} + \lvert \underset{\sim}{a} \rvert \sin\theta\, \underset{\sim}{j}$ $= 6\cos(50^\circ)\, \underset{\sim}{i} + 6\sin(50^\circ)\, \underset{\sim}{j}$ $= 3.86\underset{\sim}{i} + 4.60\underset{\sim}{j}$	**Award 2** marks for the correct solution. **Award 1** mark for substantial progress towards the solution.
(e) **ME12-2**	Find the magnitude of the vector: $\lvert \underset{\sim}{c} \rvert = \sqrt{3^2 + (-6)^2} = \sqrt{45} = 3\sqrt{5}$ Find the unit vector: $\hat{\underset{\sim}{c}} = \frac{1}{3\sqrt{5}}\left(3\underset{\sim}{i} - 6\underset{\sim}{j}\right) = \frac{\sqrt{5}}{5}\left(\underset{\sim}{i} - 2\underset{\sim}{j}\right)$ Multiply the unit vector in the direction required. $\underset{\sim}{d} = \frac{5\sqrt{5}}{5}\left(\underset{\sim}{i} - 2\underset{\sim}{j}\right) = \sqrt{5}\left(\underset{\sim}{i} - 2\underset{\sim}{j}\right)$	**Award 2** marks for the correct solution. **Award 1** mark for substantial progress towards the solution.
(f) (i) **ME12-2**	The unit vector $\underset{\sim}{i}$ lies along the x-axis, whilst the unit vector $\underset{\sim}{j}$ lies along the y-axis. As such, the two unit vectors are perpendicular, resulting in the dot product of $\underset{\sim}{i} \cdot \underset{\sim}{j} = 0$	**1 mark** if identified the vectors are perpendicular.
(ii) **ME12-2**	$\overrightarrow{AB} = (4,7) - (2,-4) = (-6,11)$ $\overrightarrow{CD} = (12,10) - (-6,-7) = (18,17)$ $Cos\theta = \frac{\overrightarrow{AB} \cdot \overrightarrow{CD}}{\lvert\overrightarrow{AB}\rvert\lvert\overrightarrow{CD}\rvert}$ $Cos\theta = \frac{-6(18) + 11(17)}{\left(\sqrt{(-6)^2 + 11^2}\right)\left(\sqrt{18^2 + 17^2}\right)}$ $\theta = 75^\circ$ (nearest degree)	**Award 3** marks for the correct solution. **Award 2** marks for substantial progress towards the correct solution. **Award 1** mark for limited progress towards solution.

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<tr><td colspan="3">ME12-3 applies advanced concepts and techniques in simplifying expressions involving compound angles and solving trigonometric equations
ME12-5 applies appropriate statistical processes to present, analyse and interpret data.</td></tr>
<tr><th>Outcome</th><th>Solutions</th><th>Marking Guidelines</th></tr>
<tr><td>(a)
ME12-5</td><td>(a)(i) $P(\text{correct}) = \frac{1}{4}$ $P(\text{incorrect}) = \frac{3}{4}$ $n = 10$
$$P(90\%) = P(\text{exactly 9 correct})$$ $$= {}^{10}C_9\left(\frac{3}{4}\right)^1\left(\frac{1}{4}\right)^9$$ $$= \frac{15}{524288}$$
(ii)
$$P(\text{At most } 90\%) = 1 - P(100\%)$$ $$= 1 - {}^{10}C_{10}\left(\frac{1}{4}\right)^{10}$$ $$= 1 - \left(\frac{1}{4}\right)^{10} = 1 - \frac{1}{1048576}$$</td><td>(a)(i)
1 mark: Correct Numerical Expression.

(ii) 2 marks: Identifying complementary relationship and correct numerical expression with $^{10}C_{10}$ value excluded.
1 mark: One of the above components.</td></tr>
<tr><td>(b)
ME12-5</td><td>(b)
$$P(\geq 5 \text{ out of } 7)$$ $$= {}^{7}C_5(0\cdot85)^5(0\cdot15)^2 + {}^{7}C_6(0\cdot85)^6(0\cdot15)^1 + {}^{7}C_7(0\cdot85)^7$$ $$= 21(0\cdot85)^5(0\cdot15)^2 + 7(0\cdot85)^6(0\cdot15)^1 + (0\cdot85)^7$$ $$\approx 0\cdot9262$$</td><td>(b) 2 marks: Correct numerical expression.
1 mark: CNE for at least the case for exactly 5 out of 7..
.</td></tr>
<tr><td>(c)
ME12-3</td><td>(c)
$$\sin\left(2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right) = \sin\left(2\left(\frac{\pi}{6}\right)\right)$$ $$= \sin\left(\frac{\pi}{3}\right)$$ $$= \frac{\sqrt{3}}{2}$$</td><td>(c)
1 mark: Correct solution</td></tr>
<tr><td>(d)
ME12-3</td><td>(d)
Let $\sqrt{3}\cos x + \sin x = R\sin(x+\alpha)$
Hence there is no vertical shift, and the amplitude will give the maximum value.
$R^2 = \sqrt{3}^2 + 1^2 = 4$
So maximum value $= R^+ = 2$.</td><td>(d) 2 marks: Correct solution with explanation/working.
1 mark: Partial solution..</td></tr>
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<tr>
<td>(e)
ME12-3</td>
<td>(e)
$\cos x - \sin x = 1$
Let $t = \tan\frac{x}{2}$ $\quad (x = \pi \text{ is not a solution})$
$\therefore \frac{1-t^2}{1+t^2} - \frac{2t}{1+t^2} = 1$
$1 - t^2 - 2t = 1 + t^2$
$0 = 2t^2 + 2t = 2t(t+1)$
$\therefore \tan\frac{x}{2} = t = 0, -1$
$\frac{x}{2} = 0, \pi, \frac{3\pi}{4} \quad \rightarrow \quad x = 0, 2\pi, \frac{3\pi}{2}$</td>
<td>(e) 3 marks: Complete solution in radians.
2 marks: Significant progress.
1 mark: Some relevant progress.

Note: There was no mark allocated to checking for pi as a possible solution, so many who omitted this step were lucky to gain full marks.</td>
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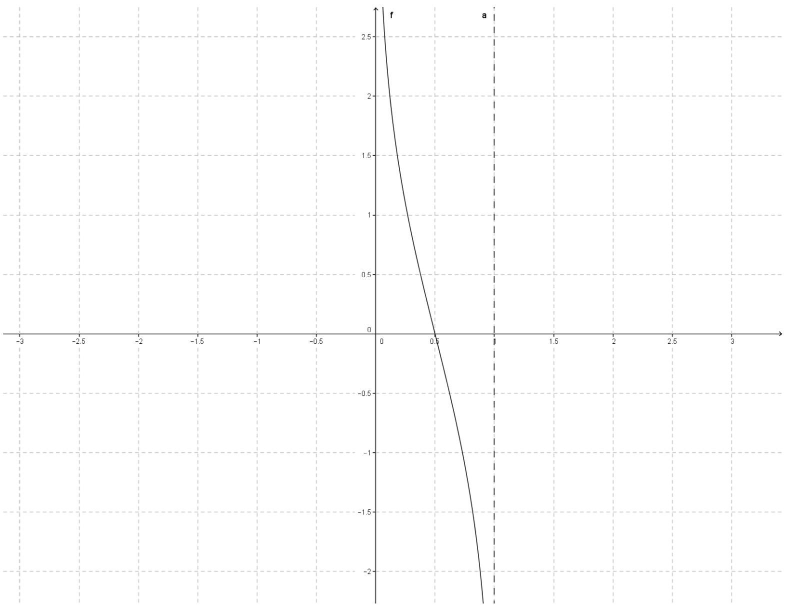
<tr>
<td>(f)
ME12-3</td>
<td>(f)(i) Find $y = f^{-1}(x)$
$x = \frac{e^{-y}}{1+e^{-y}}$
$x + xe^{-y} = e^{-y}$
$e^{-y}(x-1) = -x$
$e^{-y} = \frac{x}{1-x}$
$y = \ln\left(\frac{1-x}{x}\right)$
(ii)
</td>
<td>(f)(i) 2 marks: Simplified solution with y as the subject.
1 mark: Significant progress after switching x, and y.

(ii) 2 marks: Correct indication of shape, asymptotes, and x-intercept.
1 mark: Partial solution.</td>
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</table>

Question No. 14	Solutions and Marking Guidelines	
Outcomes Addressed in this Question		
ME12-4 uses calculus in the solution of applied problems, including differential equations and volumes of solids of revolution		
Q14 part	**Solutions**	**Marking Guidelines**
(a)	$\int_0^1 \frac{t}{\sqrt{t+1}}\,dt \qquad u=t+1 \Rightarrow t=u-1$ $=\int_1^2 \frac{u-1}{\sqrt{u}}\,du \qquad \frac{du}{dt}=1 \Rightarrow dt=du$ $=\int_1^2 \frac{u}{\sqrt{u}}-\frac{1}{\sqrt{u}}\,du \qquad t=0,\ u=1$ $t=1,\ u=2$ $=\int_1^2 u^{\frac{1}{2}}-u^{-\frac{1}{2}}\,du$ $=\left[\frac{2}{3}u^{\frac{3}{2}}-2u^{\frac{1}{2}}\right]_1^2$ $=\left[\frac{2}{3}u\sqrt{u}-2\sqrt{u}\right]_1^2$ $=\left(\frac{2}{3}(2)\sqrt{(2)}-2\sqrt{(2)}\right)-\left(\frac{2}{3}(1)\sqrt{(1)}-2\sqrt{(1)}\right)$ $=\frac{-2\sqrt{3}}{3}+\frac{4}{3}$	**(a)** **3 marks:** Complete evaluation of integral. **2 marks:** Significant progress. **1 mark:** Some relevant progress.
b)	$\frac{dy}{dx}=\frac{\cos^2 y}{e^x}$ $\frac{dy}{\cos^2 y}=\frac{dx}{e^x}$ $\int \sec^2 y\,dy=\int e^{-x}dx$ $\tan y=-e^{-x}+c$ $x=0,\ y=\frac{\pi}{4}$ $\tan\left(\frac{\pi}{4}\right)=-e^{-(0)}+c$ $1=-1+c$ $c=2$ $\therefore\ \tan y=-e^{-x}+2$ $y=\tan^{-1}\left(2-e^{-x}\right)$	**(b) 3 marks:** Complete and correct solution to the differential equation. **2 marks:** Significant progress. **1 mark:** Some relevant progress.

(c)	$y=\sqrt{m-3x}$ $x=0,\ \ y=\sqrt{m}$ $m-3x=y^2$ $x=\dfrac{m-y^2}{3}$ $V=\pi\int_0^{\sqrt{m}} x^2\ dy$ $=\pi\int_0^{\sqrt{m}}\left(\dfrac{m-y^2}{3}\right)^2\ dy$ $=\pi\int_0^{\sqrt{m}}\dfrac{m^2-2my^2+y^4}{9}\ dy$ $\dfrac{5000\pi}{27}=\ \pi\int_0^{\sqrt{m}}\dfrac{m^2-2my^2+y^4}{9}\ dy$ $=\dfrac{\pi}{9}\left[m^2y-\dfrac{2my^3}{3}+\dfrac{y^5}{5}\right]_0^{\sqrt{m}}$ $=\dfrac{\pi}{9}\left[m^2\sqrt{m}-\dfrac{2m^2\sqrt{m}}{3}+\dfrac{m^2\sqrt{m}}{5}\right]$ $=\dfrac{\pi}{9}\times\dfrac{8m^2\sqrt{m}}{15}$ $m^2\sqrt{m}\ =3125$ $m^{\frac{5}{2}}\ =3125$ $\therefore m\ =25$	**(b) 4 marks:** Complete and correct solution for m. **3 marks:** Significant progress towards finding m. **2 mark:** **1 mark:** Some relevant progress.
d) **i)**	$LHS=\dfrac{d}{dx}\left(-\cot x\right)=\dfrac{d}{dx}\left(-\dfrac{\cos x}{\sin x}\right)$ $=\dfrac{\sin x\times\left(-\sin x\right)-\cos x\times\sin x}{\sin^2 x}$ $=\dfrac{1}{\sin^2 x}=RHS$	**(d) (i)** **2 marks:** Correct differentiation **1 mark:** some progress towards correct differentiation .

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<td>d)
ii)</td>
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$$\int_{2}^{2\sqrt{3}} \frac{1}{x^2\sqrt{16-x^2}}\,dx = \frac{\sqrt{3}}{24}$$

$$x^2\sqrt{16-x^2}$$
$$= 16\sin^2\theta\left(16-16\sin^2\theta\right)^{\frac{1}{2}}$$
$$= 16\sin^2\theta 4\cos\theta$$

$$x = 4\sin\theta$$
$$\frac{dx}{d\theta} = 4\cos\theta$$
$$dx = 4\cos\theta d\theta$$

$4\sin\theta = 2$ $4\sin\theta = 2\sqrt{3}$

$\sin\theta = \frac{1}{2}$ $\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{6}$ $\theta = \frac{\pi}{3}$

$$\int_{2}^{2\sqrt{3}} \frac{1}{x^2\sqrt{16-x^2}}\,dx$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{4\cos\theta}{\left(4\sin\theta\right)^2\sqrt{16-\left(4\sin\theta\right)^2}}\,d\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{4\cos\theta}{\left(16\sin^2\theta\right)\sqrt{16-16\sin^2\theta}}\,d\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{4\cos\theta}{\left(16\sin^2\theta\right)4\cos\theta}\,d\theta$$

$$= \frac{1}{16}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\sin^2\theta}\,d\theta$$

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(d) (ii)
3 marks: Complete and correct solution
2 marks: Significant progress.
1 mark: Some relevant progress.

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$$= \frac{1}{16}\left[-\cot\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} \text{ (using part } i\text{)}$$

$$= -\frac{1}{16}\left[\frac{1}{\tan\frac{\pi}{3}}+\frac{1}{\tan\frac{\pi}{6}}\right]$$

$$= -\frac{1}{16}\left[\frac{1}{\sqrt{3}}-\sqrt{3}\right]$$

$$= \frac{2\sqrt{3}}{48}$$

$$= \frac{\sqrt{3}}{24}$$